Baryon form factors in a Contact Interaction approach to QCD.

David Wilson

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Twin Approaches to Confinement Physics,
Thomas Jefferson National Accelerator Facility.

Argonne NATIONAL LABORATORY

Work in collaboration with A. Bashir, I. C. Clöet, L. Chang and C.D. Roberts.

Strongly-coupled QCD and Hadron Physics

Connecting theory and experiment

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}^a \left(i\not{D}_{ab} - m\right)\psi^b$$
$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{2\xi}\left(\partial_\mu A^\mu_a\right)^2$$
$$+\left(\partial^\mu \bar{c}_a\right) D^{ab}_\mu c_b$$

π, K, σ η, a_1, ...
n, p, $N^{\frac{1}{2}^+}(1440)$,
$N^{12^-}(1535)$, Δ, ...
Masses, form factors ...

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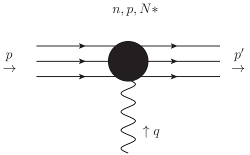
Masses, form factors ...

- Lattice QCD
- AdS/CFT
- **SDE-BSE**

- Confinement
- Dynamical chiral symmetry breaking

Roper resonance

- First observed in 1964.
- $\mathrm{N}(1440)\mathrm{P}_{11}$, $\mathrm{J}^{\mathrm{P}} = 1/2^{+}$.
- Same spin and parity as the proton.
- Has a lower mass than $\mathrm{N}(1535)S_{11}$, $\mathrm{J}^{\mathrm{P}} = 1/2^{-}$ resonance
- proton-Roper EM form factors are experimentally measurable.
- At JLab, proton-Roper transition form factors have recently been obtained, Phys. Rev. C79, 065206 (2009), C80, 055203 (2009).
- We aim to provide a theoretical calculation in a symmetry preserving framework closely connected to QCD.



Outline

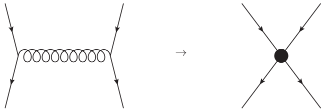
- We use a contact interaction treatment of the gluon interaction in the QCD Schwinger-Dyson equations.

→ Simple to use; generally applicable.

→ Good approximation at small q^2, especially for ground states.

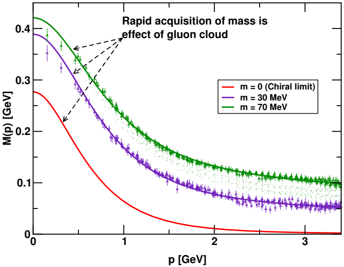
- We can calculate nucleon, excited state and transition form factors.
- Simple to generalise to any reasonably low-lying hadron.

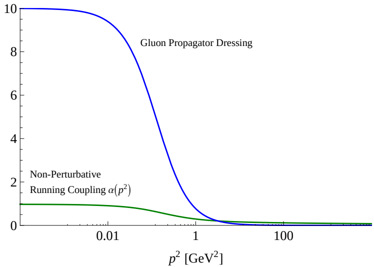
Gluon contact interaction:



Different interactions

Non-perturbative effects of **full QCD**:





Contact interactions have had phenomenological success however they neglect this running, m_q = constant. A reasonable approximation for small q^2.

Long term goals:

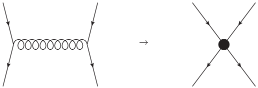
- **Calculate and compare** a **full QCD** treatment with running masses and SDE dressings **to simpler interactions**.

$\rightarrow$ **Identify the experimental signatures of full QCD**.

SDE and Bound State solutions from a contact interaction

- The effective gluon mass in the IR from motivates a truncation:
  Replace full gluon propagator,

$$\mathcal{D}_{\mu\nu}(p) = \Delta_{G\ell}(p)\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$
$$\rightarrow g_{eff}\frac{g_{\mu\nu}}{m_g^2}.$$



$\rightarrow$ Momentum independent.

- In practical terms:

$\rightarrow$ Simplifies or removes loop integrations.

$\rightarrow$ Simplifies tensor contractions.

$\rightarrow$ Like all good approximations, it makes many hard problems solvable.

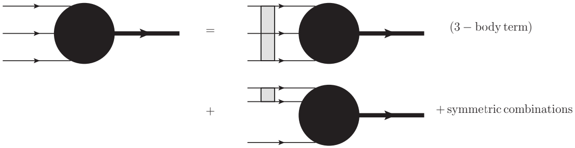
- All mass functions constant:

  $\rightarrow$ It is useful to investigate the implications of this.

  $\rightarrow$ It can be a starting point for a more advanced treatment.

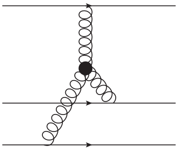
Nucleon Diquark Model

(See also talk of G. Eichmann)



- Strong evidence that diquark correlations are strong within the nucleon.
- More advanced treatments find good agreement using this model for low-lying states.
- Three body forces certainly exist but are subdominant.

$\rightarrow$ We neglect 3-body forces at this level.



Diquarks

- Mesons and diquarks are obtained from very similar Bethe-Salpeter equations.
- Each meson has diquark partner.
- Non-pointlike: finite radial extent, comparable to mesons.

Meson	Diquark
π	$(qq)^{0^+}$
ρ	$(qq)^{1^+}$
σ	$(qq)^{0^-}$
a_1	$(qq)^{1^-}$

In order to build a positive parity nucleon we use the positive parity diquarks that are related to the ρ and π.

For the $N(\frac{1}{2}^-)$ states the diquarks related to the σ and a_1 are the required contributions.

Model features:

- No dynamical gluons: Nucleon is represented as a three quark state.
- Two of the quarks assumed to be bound into a diquark correlation.
- Relevant diquarks are the 0^+ and 1^+ (qq) configurations.

→ Need positive parity diquarks for a positive parity nucleon.

→ These are simply related to the π and ρ mesons.

Diquarks:

- Already studied in this model using their Bethe-Salpeter equations.

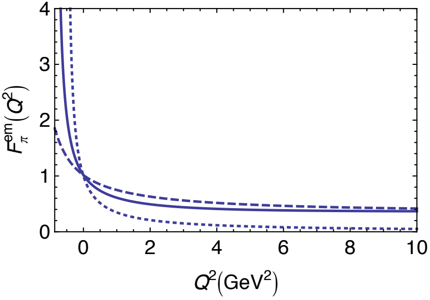
→ $m_{0^+} = 0.78$ GeV

→ $m_{1^+} = 1.06$ GeV

- Radii are comparable to their mesonic partners.
- Their EM form factors are required and have been obtained using this model.

Meson and Diquark form factors

Eg. π form factor:

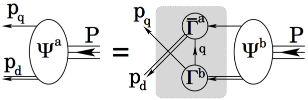


Solid curve: This model.

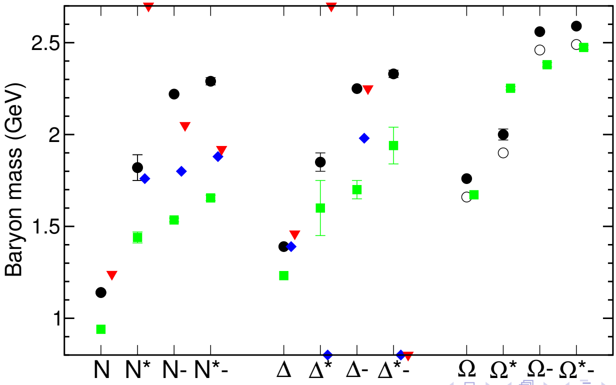
Dotted curve: Full theory.

- General feature of this model that form factors are too hard.
- This is due to the lack of running of the interaction and the mass functions.

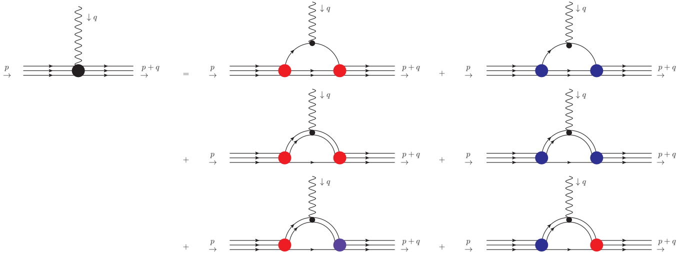
Faddeev equation Baryon masses



Masses are higher than experiment:
No pion cloud effect is accounted for in this model.
Coupling to hadronic decay channels is expected to reduce the masses.



Diagrams to calculate



- Just two diquarks in the simplest model:

1. 0^+ (diquark partner of π)
2. 1^+ (diquark partner of ρ meson)

- Photon may hit quark or diquark (4 diagrams).
- Photon may induce a diquark transition for certain quark configurations (2 diagrams).

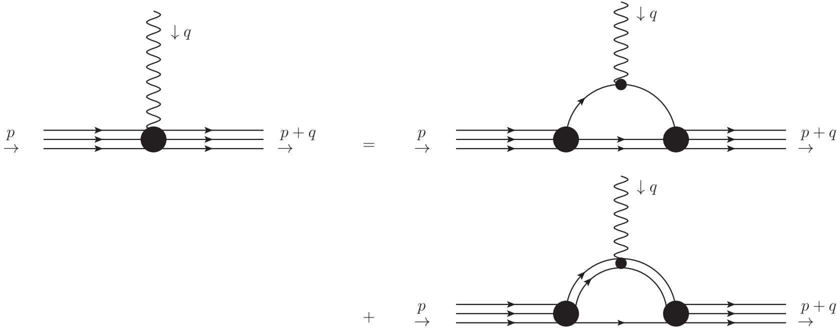
$\rightarrow$ 6 diagrams to calculate. First: scalar diagrams...

Scalar part only

Bethe-Salpeter Amplitude:

$$\Lambda_\mu^{sq} = s^2 \Lambda_+(p') \int \frac{d^4\ell}{(2\pi)^4} \left(S(\ell + p')\Gamma_\mu^{\perp}(q) S(\ell + p)\Delta(-\ell) \right) \Lambda_+(p)$$

$$\Lambda_\mu^{sd} = s^2 \Lambda_+(p') \int \frac{d^4\ell}{(2\pi)^4} \left(\Delta(k_2)\Gamma_\mu^{0^+}(q) \Delta(k_1) S(\ell) \right) \Lambda_+(p)$$



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- Just two diagrams when considering just the scalar diquark.
- Constant quark mass from solving contact gap eq., $M_q = 0.37$ GeV:

$$S(p)^{-1} = i\gamma.p + M_q$$

$\rightarrow$ Useful check: Ward Identity $\Lambda_\mu^{sq}(0) = \Lambda_\mu^{sd}(0)$

Extracting Form Factors

Elastic:

$$J_\mu(q) = ie\bar{u}(p') \left\{ \left(\gamma_\mu - \frac{\gamma.q\, q_\mu}{q^2} \right) F_1(q^2) + \frac{iq^\nu \sigma_{\mu\nu}}{2m_N} F_2(q^2) \right\} u(p)$$

Simply related to the Sachs form factors:

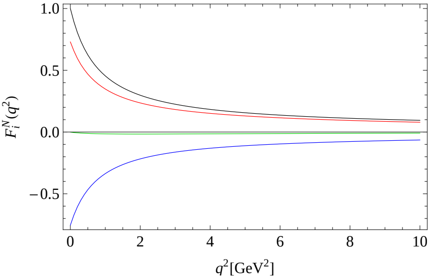
$$G_E^N(q^2) = F_1^N(q^2) - \frac{q^2}{4m_N^2} F_2^N(q^2)$$
$$G_M^N(q^2) = F_1^N(q^2) + F_2^N(q^2)$$

Transition form factor just a simple generalisation:

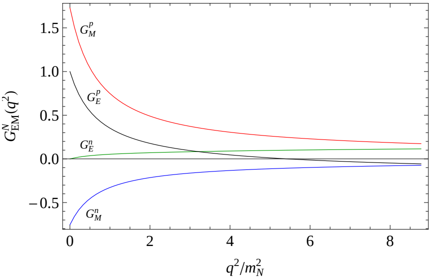
$$J^*_\mu(q) = ie\bar{u}_R(p') \left\{ \left(\gamma_\mu - \frac{\gamma.q\, q_\mu}{q^2} \right) F^*_1(q^2) + \frac{iq^\nu \sigma_{\mu\nu}}{m_N + m_R} F^*_2(q^2) \right\} u_N(p)$$

Scalar-only ground state form factors

Dirac-Pauli

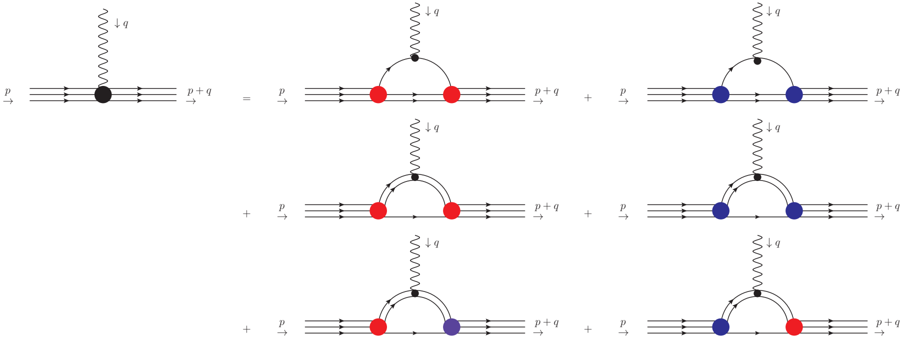


Sachs



Including the axial component

- 2 additional diagrams similar to the scalar only system.
- Additionally have a scalar-axial diquark transition diagram (also axial-scalar transition)
- Relative strength of components comes from Faddeev solution.



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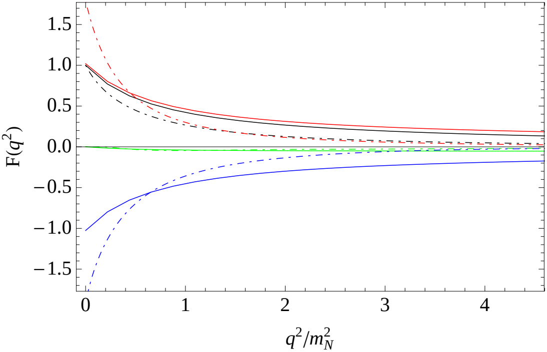
Components:

- Scalar diquark: [ud]u in proton, [ud]d in neutron
- Axial diquarks: {uu}d and [ud]u in proton,
  {dd}u and [ud]d in neutron.
- These come with different isospin and charge factors.

Ground state composition: (Scalar, Axial)=(0.88, 0.47)

Elastic Ground state form factors



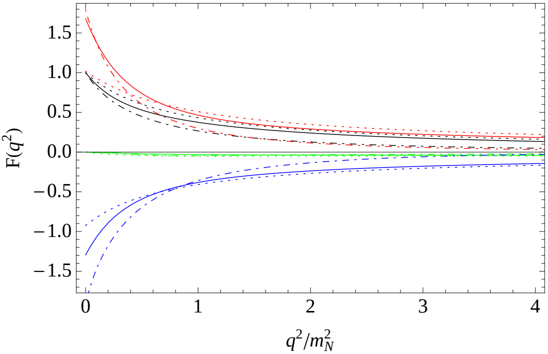
Solid: This calculation,

Dot-dashed: Curve Fitted to Experimental data.

Black: F_1^p, Red: F_2^p, Green: F_1^n, Blue: F_2^n.

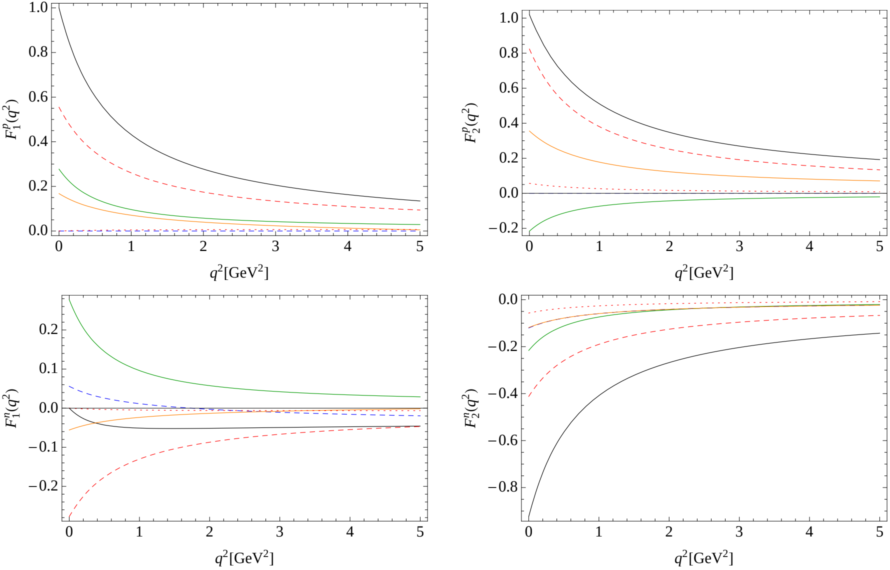
- Form factors are too hard $\rightarrow$ more like a pointlike composite state.
- Magnetic moments are too small $\rightarrow$ can be improved by using an extended quark-photon vertex.

Elastic Ground state form factors with AMM vertex



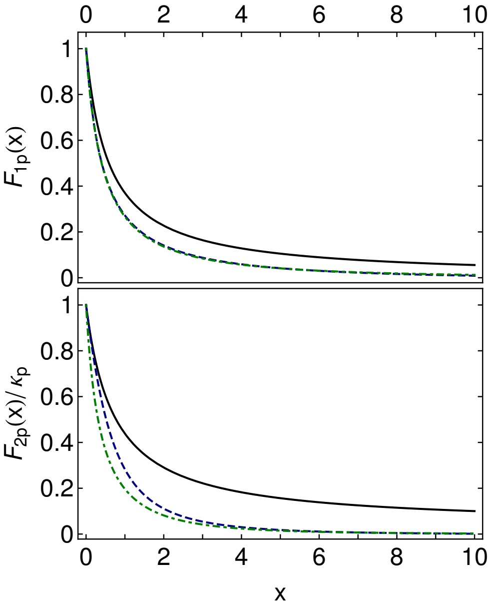
Solid: AMM vertex, Dotted: Previous result,
Dot-dashed: Curve Fitted to Experimental data.
Black: F_1^p, Red: F_2^p, Green: F_1^n, Blue: F_2^n.

Ground state form factor term-by-term



→ All diagrams are important.

Plotted again...



- $x = q^2/m_N^2$ - removes some of the pion cloud effect.
- F_{2p} also normalised to unity at $x = 0$.
- Reasonable representation at small x, but already a significant deviation for $x \sim 1$.

Solid: This calculation.
Dashed: Clöet *et al* (w/ running dressing fns.)
Dot-dashed: Experimental fit.

Ground state and Excited state comparison

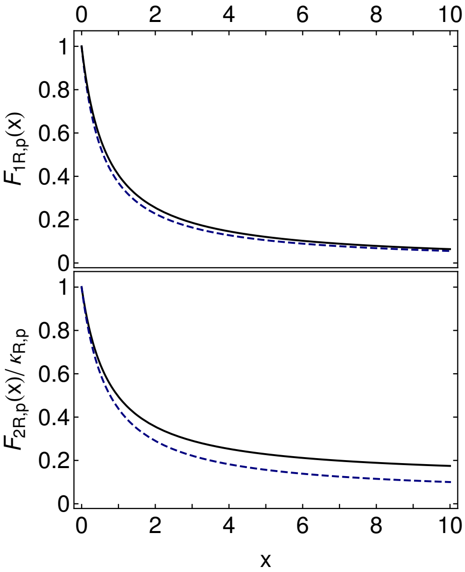
- Using a simple Faddeev equation in the same contact interaction framework, we can insert a node into the integrand in order to produce an excited state.
- The excited state and ground state satisfy an orthogonality condition.

Faddeev solution components:

	Scalar	Axial γ_μ	Axial p_μ
N	0.88	-0.38	-0.07
N*	-0.44	-0.03	0.73

- Excited state mass: $m_{N^*} = 1.73$ GeV.
- Orthogonality is very important and results in an overall zero at $q^2 = 0$ in the transition.
- Overall sign does not affect elastic form factors.
- Roper more axial than scalar; opposite to ground state.

Excited state form factors - charged

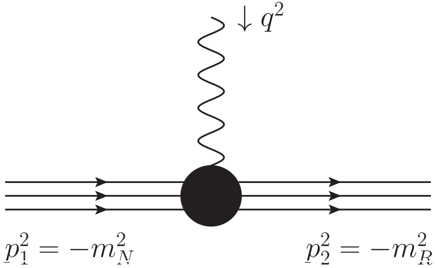


Solid: Excited state, Dashed: Ground state.

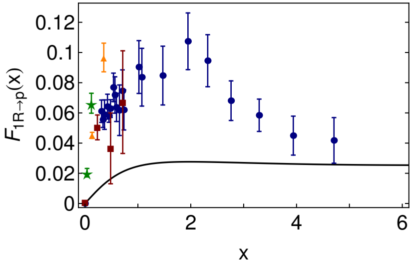
- $x = p^2/m_B^2$
- F_1 very similar.
- $\kappa_{R,p} = 0.59$ in this calculation.
- It is likely this is too small in magnitude.

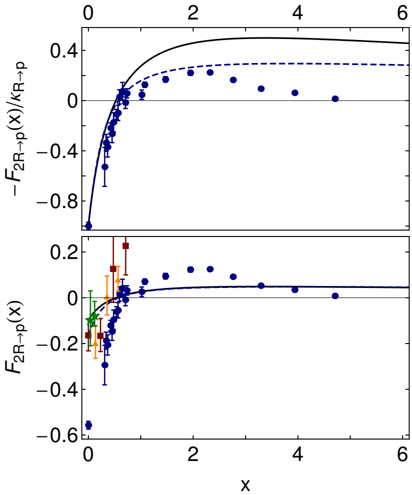
Transition form factors

Now: Roper $\rightarrow$ nucleon EM transition:



Transition form factors





- Negative at small x due to orthogonality

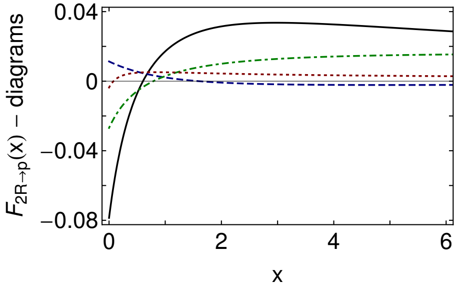
→ Scalar diquark components have opposite signs.

- F_2 zero is quite generic.
- Related to the zero in the Roper's Faddeev amplitude.

→ Typical of a radial excitation.

Diagram contributions

- There is a zero in each term so it is natural that a zero arises.
- Orthgonality makes the scalar diquark contribution negative at $x = 0$.

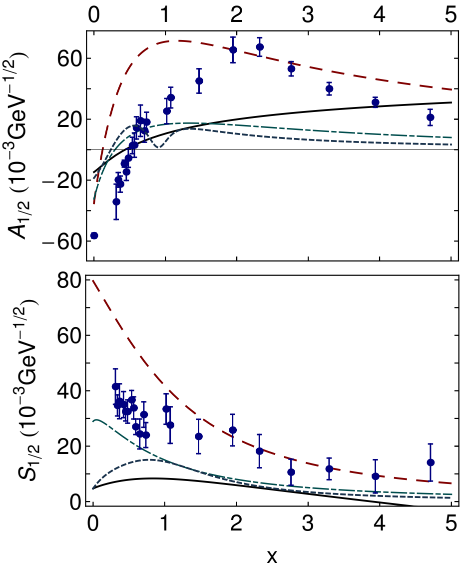


Solid: photon hits u-quark, scalar diquark spectator, Dashed: photon on scalar diquark,

Dot-dashed: photon on axial vector diquark, Dotted: photon-quark with AV spectator.

	Scalar	Axial γ_μ	Axial p_μ
N	0.88	-0.38	-0.07
N^*	-0.44	-0.03	0.73

Considering the meson cloud



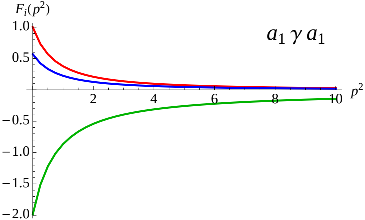
Plotted using helicity amplitudes $S_{1/2}$, $A_{1/2}$.

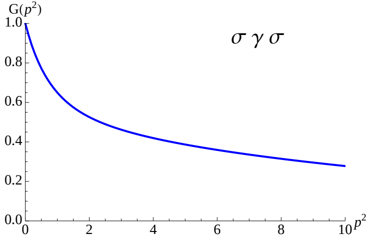
Solid: this calculation
Short dash: EBAC
Dash-dot: CQM
(Cardarelli et al, PLB397, 13 (1997))
Long dash: LFCQM
(I. G. Aznauryan, PRC76, 025212 (2007))

- Closest agreement with EBAC bare FF at small q^2.

Next steps: $N \rightarrow N^{\frac{1}{2}^-}(1535)$

- Requires EM elastic and transition diquark form factors for all possible combinations.
- Partners of: $a_1\gamma a_1$, $a_1\gamma\rho$, $a_1\gamma\pi$, $\sigma\gamma\sigma$, $\sigma\gamma\rho$, ...
- All are straightforwardly calculable in the contact interaction model framework, eg:





$$\Lambda^{a_1}_{\mu\nu\rho} = \sum_i F^{a_1}_i(p^2)T_{i,\mu\nu\rho}(p) \qquad \Gamma^{\sigma}_{\mu}(p) = 2p_\mu G^{\sigma}(p^2)$$

Transition calculation quite simple since it may only occur via a diquark transition diagram.

Conclusions

- Nucleon, Roper and Transition form factors have been calculated in a contact interaction model.
- Similar to other applications of this model, we find:

→ Form factors typically harder than experiment or full QCD.

→ F_2 form factors typically too small in magnitude.

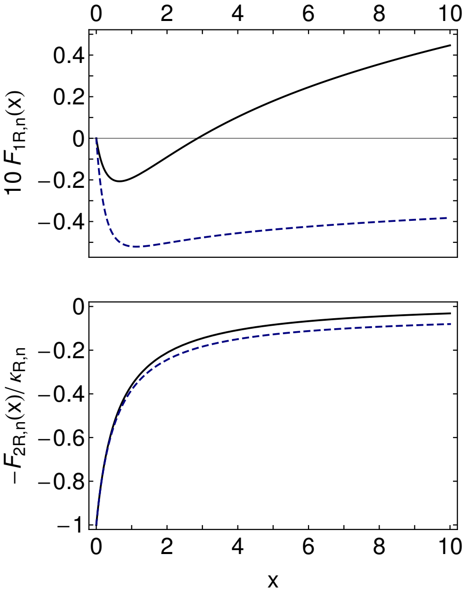
→ This is well understood and due to effects of the quark-photon vertex and coupling to the virtual meson cloud.

- No calculation currently agrees with the data

→ They shouldn't because no calculation includes the meson cloud.

- We essentially agree with EBAC's bare form factors.
- Our work adds to growing body of evidence that it is the Bare Form Factors with which structure calculations should compare.
- A calculation using a full interaction is certainly required, this will follow after a study of the $1/2^-$ parity partner using this interaction.

Excited state form factors - neutral



Solid: Excited state, Dashed: Ground state.

- F_2 very similar in this case.
- F_1 exhibits a zero, possibly due to FF diagrams being too hard

Integral Regularisation

$$\frac{1}{p^2 + M^2} = \int_0^\infty d\tau\, e^{-\tau(p^2+M^2)} \to \int_{\tau_{UV}^2}^{\tau_{IR}^2} d\tau\, e^{-\tau(p^2+M^2)}$$
$$= \frac{e^{-\tau_{UV}^2(p^2+M^2)} - e^{-\tau_{IR}^2(p^2+M^2)}}{p^2 + M^2}$$

- τ parameters universal. Fixed by studies of meson masses and gap eq.
- Gives confinement.
- Regulates the UV divergences due to point-like interaction.

Use on-shell form factors

- Require form factor inputs for diquarks.
- Have only been calculated on-shell.
- Approximate form factor by on-shell contribution.
- Dominant part of integral should come from on-shell region.